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APPLICATIONS OF LIGHT-FRONT DYNAMICS IN QCD

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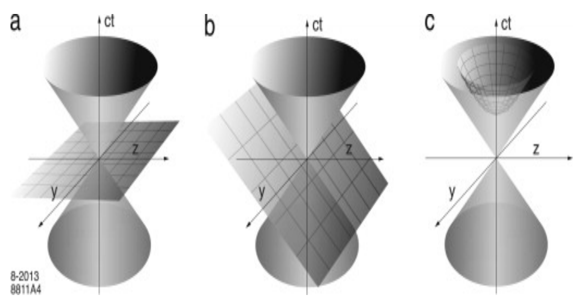
ABSTRACT:

Intuitive account of Deep Inelastic Scattering (DIS) can be understand form the Light-front formulation of QCD by keeping close contact with the parton ideas. Later on one obtains the expected scaling violations and many other interesting results by the application of Light-Front Dynamics.

KEY WORDS: *Light-front formulation , Light-Front Dynamics.*



1. INTRODUCTION:

Paul Dirac in 1949 introduced a coordinate system to describe the geometry of Lorentz transformations. It uses a rectangular coordinate system commonly known as the light-cone coordinate system. He proposed Light front **[1]** as the two-dimensional surface in space-time formed by a plane wave front advancing with the velocity of light. For example $x^+ = 0$ plane in **fig.1** gives a light-front. Light-front dynamics is the description of the development of a relativistic system along a light-front direction.

Features of a dynamical system is studied by ten fundamental quantities these are energy, momentum, angular momentum, and boost. In the Hamiltonian form of dynamics, the dynamical variables are supposed to deal with the physical conditions at some instant of time, for example $x^0 = 0$. Alternatively, Dirac proposed other forms of relativistic dynamics in which the dynamical variables refer to physical conditions on a front $x^+ = 0$ (surface tangential to the light cone). Eventually such dynamics is called light-front dynamics.

Why do we need such coordinates which depend on a particular choice of the x^3 axis?

Answer is such coordinates transform easily under boosts along the x^3 axis and when a vector is advances speedily along the x^3 axis, light-cone coordinates gives information about the large and small components of momentum. Customarily light-cone coordinates uses to express process like high energy hadron scattering. In that situation, there is a natural choice of an axis, the collision axis, and one frequently needs to transform between different frames related by boosts along the axis. Commonly used frames include the rest frame of one of the incoming particles, the overall center-of-mass frame, and the center-of-mass of a partonic subprocess. Evolution in light-front time x^+ has extraordinary advantages for relativistic systems, arise from the fact that seven out of the ten Poincare generators, including a Lorentz boost, are kinematical (interaction independent) when one quantizes a theory at fixed light-front time **[2]**.

2. LIGHT CONE COORDINATES:

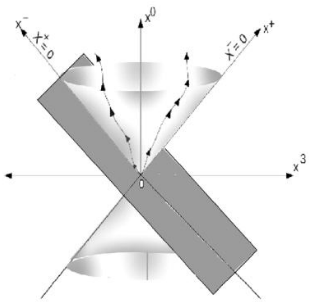


Fig. : A space-time diagram with orthogonal axes x^+ and x^-, $x^\pm = 0$ are the light-cone axes. The curves with arrows are possible world-line of physical particle

Consider a linear combinations of the time coordinate and one spatial coordinate (say x^3) as $x^+ = x^0 + x^3$, $x^- = x^0 - x^3$. The new coordinates x^+ and x^- are called light-cone coordinates because the associated coordinate axes are the world-lines for beams of light emitted from the origin along the x^3 axis. Thus, the complete set of light-cone coordinates is written for the four vector as

$x^\mu \equiv (x^+ , x^- , x^1 , x^2) = (x^+ , x^- , x^\perp)$

where

$x^\perp = (x^1 , x^2)$

for transverse variable.

For a beam of light going in the positive x^3 direction, we have $x^3 = ct = x^0$, and thus $x^- = 0$ (line on x^+ axis).
For a beam of light going in the negative x^3 direction, we have $x^3 = -ct = -x^0$, and thus $x^+ = 0$ (line on x^- axis).
The $x^\pm$ axes are lines at 45^o with respect to the x^0, x^3 axes.

3. LIGHT CONE TIME:

Instead of x^0, both x^+ and x^- can act as time, in fact, both have equal right to be called a time coordinate, although in the standard sense of the word neither one is a time coordinate. Light-cone time is not quite the same as ordinary time. Perhaps the most familiar property of time is that it goes forward for any physical motion of a particle. We choose x^- and x^+ as light-cone position and light-cone time respectively. Physical motion starting at the origin is represented in fig.1 as curves that remain within the light cone and whose slopes never go below 45^o . For all these curves, as we follow the arrows, both x^+ and x^- increase. Taking differentials of x^+ and x^- we readily find

$dx^+ dx^- = (dx^0)^2 - (dx^3)^2$

and we can write the invariant interval expressed in terms of the light-cone coordinates as:

$$ds^2 = (dx^0)^2 - (dx^3)^2 - (dx^1)^2 - (dx^2)^2 = dx^+dx^- - (dx^\perp)^2$$

4. LIGHT CONE VELOCITY:

Consider any Lorentz vector a^μ , having light-cone components $a^\pm = a^0 \pm a^3$. The scalar product between these two Lorentz vectors is

$$a \cdot b = a^{\mu} \cdot b^{\mu} = a^0 b^0 - a^3 b^3 - a^{\perp} b^{\perp} = \frac{1}{2} a^+ b^- + \frac{1}{2} a^- b^+ - a^{\perp}.b^{\perp} = g_{\mu\nu} a^{\mu} b^{\nu}$$

Where $g_{\mu\nu}$ is the metric tensors given by

$$g_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Indices μ, ν run over four values {+, −, 1, 2}. Now consider dot product

$$\begin{aligned} a \cdot b &= a_{\mu} \cdot b^{\mu} = a_+ b^+ + a_- b^- + a_{\perp} b^{\perp} \\ &= a^{\mu} \cdot b^{\mu} = \frac{1}{2} a^- b^+ + \frac{1}{2} a^+ b^- - a^{\perp} b^{\perp} \\ \therefore \ a_+ &= \frac{1}{2} a^-, \quad a_- = \frac{1}{2} a^+ \quad a_{\perp} = -a^{\perp}. \end{aligned}$$

If we lower or raise the zeroth index of in any Lorentz frame we get an extra sign. However, in light-cone coordinates the index changes, and we get an extra term. Physics described using light-cone coordinates looks aberrant, to see this consider a particle moving along the x 3 -axis with speed parameter $\beta = {}^{v}/_{c}$.

The positions are expressed in terms of time as $x^3(t)=vt=\beta x^0$, $x^1(t)=x^2(t)=0$ and at t = 0, the positions are $x^1=x^2=x^3= 0$. This gives relation between light-cone position x^- and light-cone time x^+ as

$$x^- = \frac{1-\beta}{1+\beta} x^0 \quad \Longrightarrow \frac{dx^-}{dx^+} = \frac{1-\beta}{1+\beta} = \ say\ V_c$$

V_c is the light-cone velocity which gives some interesting cases as listed below:

• β = 1: light moving to the right, $V_c = 0$ because x−does not change at all.

• β = 0: a static particle in standard coordinates is moving quite fast in light-cone coordinates.

• V_c increases as β grows negative, and becomes infinite at β = −1.

• β = −1 case seems odd, there is no clash with relativity. Light-cone velocities are just unusual. The light-cone is a frame where kinematics has a non-relativistic flavor, and infinite velocities are possible.

• light-cone coordinates were introduced as a change of coordinates, not as a Lorentz transformation.

• There is no Lorentz transformation that takes the coordinates (x^0, x^3) into coordinates $(x'^0, x'^3) = (x^+, x^-)$.

5. FREE SCALAR FIELDS

A scalar field ϕ(t, **x**) ≡ ϕ(x) is a single real function of space-time and remain scalar under Lorentz transformations. To write Lagrangian density *L* in scalar field, we need kinetic energy density T and potential energy density V in scalar field. T is proportional to the square of the rate of change of the field with time (as in classical, it is proportional to the square of velocity)

$T = {}^{1}/_{2} (\partial_o \phi)^2$

At any fixed time, T is a function of position. The total kinetic energy will be the integral of the density T over space. Other required term to write *L* is the potential energy density. Potential energy for a simple harmonic oscillator with equilibrium position x = 0, varies proportional to x^2. Now, consider the equilibrium value of the field is $\phi = 0$ and we write the simplest form of V as

$V = {}^1/_2\, \mu^2\, \phi^2$.

Note that the constant μ introduced here has the units of mass. Comparing T and V which are dimensionally same, we can write $[\mu] = [\partial_o] = L^{-1} = M$. Thus the Lagrangian density *L* can be written as:

$$\boldsymbol{L = T - V = {}^1/_2\, (\partial_o\, \phi)^2 - {}^1/_2\, \mu^2\, \phi^2 \ldots\ldots\ldots\ldots (1)}$$

Above Lagrangian density (eq. 1) is not Lorentz invariant. Since in special relativity,the field vary in time must also associated with the the field vary in space. So the contribution to the energy should come from both the part. Hence for the missing term we will take

$V' = \frac{1}{2}\sum_i(\partial_i\phi)^2 = \frac{1}{2}(\nabla\phi)^2$ where ∂_i are derivatives with respect to spatial coordinates.

As kinetic energy is always associated with time derivatives and to keep *L* as Lorentz invariant we have written this new contribution V' to the potential term. Thus Lagrangian density now become:

$$\begin{aligned}
\mathcal{L} = T - V' - V &= \frac{1}{2}\partial_0\phi\partial_0\phi - \frac{1}{2}\partial_i\phi\partial_i\phi - \frac{1}{2}\mu^2\phi^2 \\
\xrightarrow[\phi=\phi(x^+,x^-,x^\perp)]{In\ light-front} \quad \mathcal{L} &= \frac{1}{2}\partial^+\phi\partial^-\phi - \frac{1}{2}\partial^\perp\phi.\partial^\perp\phi - \frac{1}{2}\mu^2\phi^2. \\
\xrightarrow{\text{Associated action}} \quad S &= \int d^Dx(\frac{1}{2}\partial_0\phi\partial_0\phi - \frac{1}{2}\partial_i\phi\partial_i\phi - \frac{1}{2}\mu^2\phi^2). \\
\xrightarrow[\delta\phi\ with\ \delta S=0]{Small\ Variation} \quad \delta S &= \int d^Dx(\partial_0(\delta\phi)\partial_0\phi - \partial_i(\delta\phi)\partial_i\phi - \mu^2\phi(\delta\phi)) \\
&= \int d^Dx(\delta\phi)(-\partial_0\partial_0\phi + \partial_i\partial_i\phi - \mu^2\phi) = 0. \\
\xrightarrow{Equation.\ of\ motion} \quad & \partial_0\partial_0\phi - \partial_i\partial_i\phi + \mu^2\phi = 0 \ \ (Klien\ Gordon\ eq.) \\
& \partial_0\partial_0\phi - \partial_3\partial_3\phi - \partial_1\partial_1\phi - \partial_2\partial_2\phi + \mu^2\phi = 0 \\
\xrightarrow[\phi=\phi(x^+,x^-,x^\perp)]{In\ the\ light-front} \quad & (\partial^+\partial^- - \partial^\perp + \mu^2)\phi = 0.
\end{aligned}$$

Here the volume element $d^Dx = dx^0\, dx^1 \ldots . dx^d$, and D = d + 1 is the number of space time dimensions (for light-front $d^Dx = d^4x =$ $\boldsymbol{{}^1/_2}$ $dx^+\, dx^-\, d^2x^\perp$) and index i denotes summation over 1, 2 and 3. For free field, it's equations of motion are linear. If each term in the action is quadratic in the field, as it is in above action S, the equations of motion will be linear in the field. For interacting field (not free) the action contains terms of order three or higher in the field. Taking Fourier transform the spatial dependence of the field ϕ, (changing $x^- \rightarrow p^+$, $x^\perp \rightarrow p^\perp$)

$$\phi(x^+, x^-, x^\perp) = \int \frac{dp^+}{2\pi} \int \frac{d^2p^\perp}{(2\pi)^2} e^{(-ix^-p^+ + ix^\perp\cdot p^\perp)}\phi(x^+, p^+, p^\perp)$$

Thus the equation of motion (eq,1) becomes

$$[i\partial^+ - \frac{1}{p^+}(p^\perp p^\perp + \mu^2)]\phi(x^+, p^+, p^\perp) = 0$$

This light-cone equation is a first-order differential equation in light-cone time similar to that of Schrodinger equation, which is also first order in time. This fact is useful when we study how the quantum point particle is related to the scalar field. In quantum field theory (QFT), the fields (dynamical variables) are operators. The state space in a QFT is described using a set of particle states. Let us consider a classical field configuration for $\phi(x^+, x^-, x^\perp) = \phi(x)$, the quantized free scalar field can be written as **([3], [4], [5])**

$$\phi(x) = \int_{0^+}^{\infty} \frac{dk^+ d^2k^\perp}{2k^+(2\pi)^3} \left[a(k)\, e^{-ik.x} \; + \; a^\dagger(k)\, e^{ik.x}\right]$$

6. CONCLUSION:

Coming to the application of Light-Front Dynamics, we can give a complete account of virtual Compton scattering $\gamma^* p \rightarrow p$ at small momentum transfer squared t and large initial photon virtuality Q^2 in terms of the light-cone wave functions of the target proton. Deep inelastic lepton-hadron scattering is a highly relativistic process. Feynman introduced the parton model in the infinite momentum frame which successfully described scaling of the structure functions. With this, DIS can be studied from the Light-front formulation of QCD. we present a study of Generalized Parton Distributions (GPDs) For Protons In Position Space which is mainly based on Light-Front theory **([5],[6])**.

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