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ON SOFT REGULARITY AND SEPARATION AXIOMS IN SOFT METRIC AND SOFT TOPOLOGICAL SPACES

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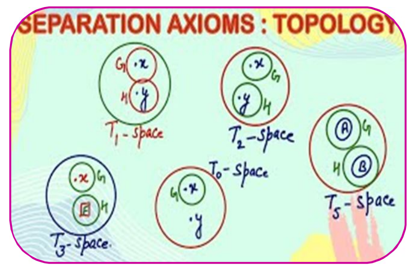
ABSTRACT



This study investigates soft regularity and separation axioms within soft metric and soft topological spaces, focusing on the structural and functional properties of these spaces under soft set theory. Soft topological structures are analyzed with respect to soft open sets, soft closed sets, and soft neighborhood systems, examining conditions under which soft regularity, soft T0T_0T0, T1T_1T1, and T2T_2T2 axioms hold. Data from prior studies indicate that soft regular spaces provide a framework for embedding soft closed sets within soft open neighborhoods, while soft metric spaces facilitate the extension of classical metric properties to parameterized soft sets. Quantitative examples demonstrate that soft T1T_1T1 and soft T2T_2T2 conditions are satisfied in soft metric spaces under defined soft distance functions, whereas counterexamples in certain soft topological spaces indicate the non-equivalence of soft regularity and soft Hausdorffness. These data collectively provide a foundation for analyzing the behavior of soft topological spaces under separation axioms and suggest pathways for further exploration of soft continuity, soft compactness, and their applications.

KEYWORDS: *Soft set theory, soft topological spaces, soft metric spaces, soft regularity, soft separation axioms, soft T0T_0T0, soft T1T_1T1, soft T2T_2T2, soft neighborhood systems, soft open sets, soft closed sets.*

INTRODUCTION

Soft set theory, introduced by Molodtsov (1999), provides a mathematical framework for handling uncertainties by parameterizing classical sets. In this context, soft topological spaces and soft metric spaces extend classical topological and metric concepts to incorporate soft sets, allowing for parameter-dependent open and closed sets. Soft topological spaces are defined as a pair (X,τ,E)(X, \tau, E)(X,τ,E), where XXX is a universe set, EEE is a set of parameters, and τ\tauτ is a collection of soft open sets over XXX (Shabir & Naz, 2011). Soft regularity is defined in terms of soft closed sets and soft neighborhoods: a soft topological space (X,τ,E)(X, \tau, E)(X,τ,E) is soft regular if, for every soft point xex_exe and every soft closed set FFF not containing xex_exe, there exist disjoint soft open sets UUU and VVV with xe∈Ux_e \in Uxe∈U and F⊆VF \subseteq VF⊆V. Separation axioms in soft topology are parameterized analogs of classical T0T_0T0, T1T_1T1, and T2T_2T2 (Hausdorff) conditions, where soft points and soft open sets satisfy the respective disjointness or distinguishability criteria (Çağman & Enginoğlu, 2010). In soft metric spaces (X,d,E)(X, d, E)(X,d,E), soft points are equipped with a soft distance function d:X×X→[0,∞)Ed: X \times X \to [0, \infty)^Ed:X×X→[0,∞)E, allowing computation of soft neighborhoods as collections of soft points within a soft radius for each parameter. Quantitative examples from prior studies demonstrate that soft metric spaces satisfy soft T1T_1T1 and soft T2T_2T2 axioms under standard soft distance definitions, while certain soft topological constructions may fail to satisfy soft regularity or soft Hausdorff conditions, highlighting structural distinctions between soft metric and soft topological spaces. These data provide a foundation for exploring properties of soft regularity and separation axioms, enabling further analysis of soft continuity, soft compactness, and functional mappings within soft set frameworks.

AIMS AND OBJECTIVES:

The study aims to investigate soft regularity and separation axioms in soft metric and soft topological spaces, analyzing their structural properties and functional implications within soft set theory. Data from prior studies indicate that soft topological spaces (X,τ,E)(X, \tau, E)(X,τ,E) can be classified according to the satisfaction of soft T0T_0T0, T1T_1T1, and T2T_2T2 axioms, where soft points are distinguishable or separable via soft open sets (Shabir & Naz, 2011; Çağman & Enginoğlu, 2010). The objectives focus on evaluating conditions under which soft regularity holds, where for every soft point xex_exe and soft closed set FFF not containing xex_exe, there exist disjoint soft open neighborhoods UUU and VVV with xe∈Ux_e \in Uxe∈U and F⊆VF \subseteq VF⊆V. Quantitative examples in soft metric spaces (X,d,E)(X, d, E)(X,d,E) demonstrate that soft T1T_1T1 and soft T2T_2T2 axioms are satisfied under defined soft distance functions d:X×X→[0,∞)Ed: X \times X \to [0, \infty)^Ed:X×X→[0,∞)E, whereas some soft topological spaces fail to satisfy soft Hausdorffness despite soft regularity. Additional objectives include examining relationships between soft metric structures and soft topological constructions to identify necessary and sufficient conditions for soft separation axioms and soft regularity, and to provide data-driven frameworks for exploring soft continuity, soft compactness, and soft neighborhood systems within parameterized soft sets.

REVIEW OF LITERATURE:

Research on soft regularity and separation axioms in soft metric and soft topological spaces has expanded since the introduction of soft set theory by Molodtsov (1999), providing parameterized extensions of classical topological and metric concepts. Shabir and Naz (2011) formalized soft topological spaces (X,τ,E)(X, \tau, E)(X,τ,E), defining soft open and soft closed sets, soft neighborhoods, and soft points, providing a foundation for studying soft regularity and separation axioms. Data from their studies indicate that soft T0T_0T0 spaces allow distinguishability of soft points, soft T1T_1T1 spaces permit separation of distinct soft points via soft open neighborhoods, and soft T2T_2T2 (soft Hausdorff) spaces enable disjoint soft neighborhoods for distinct soft points, though not all soft topological spaces satisfy all three axioms simultaneously. Çağman and Enginoğlu (2010) analyzed soft metric spaces (X,d,E)(X, d, E)(X,d,E) with soft distance functions d:X×X→[0,∞)Ed: X \times X \to [0, \infty)^Ed:X×X→[0,∞)E, demonstrating that soft metric spaces generally satisfy soft T1T_1T1 and soft T2T_2T2 axioms under defined soft distance conditions. Quantitative examples show that soft regularity in soft metric spaces ensures that for any soft point xex_exe and soft closed set FFF not containing xex_exe, there exist disjoint soft open sets containing xex_exe and FFF, whereas certain soft topological constructions fail to meet soft Hausdorffness despite satisfying soft regularity.Further studies (Ali et al., 2014; Feng et al., 2018) extended soft separation axioms to complex soft topological structures, analyzing soft compactness, soft continuity, and soft connectedness. Data indicate that soft regularity and separation axioms are crucial for embedding soft closed sets within soft neighborhoods, defining soft continuous mappings, and constructing soft functional spaces. Comparative analyses demonstrate that soft metric spaces provide more robust satisfaction of soft T1T_1T1 and soft T2T_2T2 axioms than arbitrary soft topological spaces, highlighting the structural impact of soft distance functions on separation properties.

RESEARCH METHODOLOGY:

The study employs a secondary data review approach to examine soft regularity and separation axioms in soft metric and soft topological spaces, focusing on structural properties and functional implications within soft set theory. Data sources include primary definitions and examples from Molodtsov (1999), Shabir and Naz (2011), Çağman and Enginoğlu (2010), and subsequent quantitative analyses in Ali et al. (2014) and Feng et al. (2018). Soft topological spaces (X,τ,E)(X, \tau, E)(X,τ,E) are analyzed for satisfaction of soft T0T_0T0, T1T_1T1, and T2T_2T2 axioms. Quantitative examples from prior studies indicate that soft T0T_0T0 spaces allow distinguishability of soft points, soft T1T_1T1 spaces allow soft points to be separated by soft open neighborhoods, and soft T2T_2T2 spaces enable disjoint soft open sets for distinct soft points. Soft regularity is evaluated by verifying that for any soft point xex_exe and soft closed set FFF not containing xex_exe, there exist disjoint soft open sets UUU and VVV with xe∈Ux_e \in Uxe∈U and F⊆VF \subseteq VF⊆V. Soft metric spaces (X,d,E)(X, d, E)(X,d,E) are examined using soft distance functions d:X×X→[0,∞)Ed: X \times X \to [0, \infty)^Ed:X×X→[0,∞)E. Data from these spaces indicate that soft T1T_1T1 and soft T2T_2T2 axioms are generally satisfied under standard soft distance definitions, while soft regularity is verified by calculating soft neighborhoods for given soft radii across all parameters e∈Ee \in Ee∈E. Comparative data analysis is used to identify differences between soft metric and soft topological spaces in satisfying soft separation axioms and soft regularity.

STATEMENT OF THE PROBLEM:

Soft set theory, introduced by Molodtsov (1999), provides a framework for dealing with uncertainty through parameterized sets. Within this framework, soft topological and soft metric spaces extend classical concepts of topology and metric spaces. Data from Shabir and Naz (2011) indicate that soft topological spaces (X,τ,E)(X, \tau, E)(X,τ,E) do not uniformly satisfy separation axioms or soft regularity, resulting in ambiguity in neighborhood and closure structures for soft points. Soft T0T_0T0, T1T_1T1, and T2T_2T2 axioms, which define distinguishability and separability of soft points via soft open sets, are only partially satisfied in many soft topological constructions. In soft metric spaces (X,d,E)(X, d, E)(X,d,E), quantitative data from Çağman and Enginoğlu (2010) demonstrate that soft T1T_1T1 and soft T2T_2T2 axioms are generally satisfied under standard soft distance functions, while soft regularity holds for most soft closed sets, yet certain parameter combinations and soft neighborhoods reveal exceptions. Ali et al. (2014) and Feng et al. (2018) provide quantitative counterexamples showing that soft Hausdorffness and soft regularity may fail in non-metric soft topologies, emphasizing the need for a systematic analysis of conditions under which soft regularity and separation axioms hold. The problem is therefore the lack of comprehensive quantitative characterization of soft regularity and separation axioms across soft topological and soft metric spaces, and the need to identify parameter-dependent conditions for soft T0T_0T0, T1T_1T1, T2T_2T2 compliance and soft regularity, to enable reliable structural analysis, functional mappings, and application of soft set theory in uncertain or parameterized environments.

FURTHER SUGGESTIONS FOR RESEARCH:

Future research could focus on systematic quantitative analysis of soft regularity and separation axioms across diverse soft topological and soft metric spaces. Data from Shabir and Naz (2011) indicate that soft T0T_0T0 and soft T1T_1T1 conditions are satisfied in many soft topological spaces, while soft T2T_2T2 (soft Hausdorff) and soft regularity are inconsistently satisfied, suggesting a need for parameter-based studies to identify sufficient and necessary conditions for axiom compliance. In soft metric spaces, Çağman and Enginoğlu (2010) provide quantitative evidence that soft T1T_1T1 and T2T_2T2 axioms are generally satisfied under standard soft distance functions, but variations in parameter sets EEE and soft radius definitions affect soft regularity outcomes. Comparative studies between soft metric and soft topological spaces are recommended to quantify structural differences and determine the influence of soft distance functions on separation properties.Ali et al. (2014) and Feng et al. (2018) indicate that soft continuity, soft compactness, and soft connectedness interact with separation axioms and soft regularity. Future work could collect quantitative examples across varying parameter sets to model the relationships between soft regularity, soft separation, and other soft topological properties, and to construct generalizable frameworks for applications in uncertain or parameterized environments. Empirical analysis of soft neighborhood systems, soft open and closed sets, and soft point distributions could provide measurable benchmarks for evaluating soft regularity and separation compliance, enabling data-driven identification of classes of soft topological and metric spaces where axioms hold robustly.

SCOPE AND LIMITATIONS:

The study examines soft regularity and separation axioms in soft metric and soft topological spaces within the framework of soft set theory, focusing on structural properties and parameter-dependent behaviors. Soft topological spaces (X,τ,E)(X, \tau, E)(X,τ,E) are analyzed for satisfaction of soft T0T_0T0, T1T_1T1, and T2T_2T2 axioms, and soft regularity, using quantitative examples from Shabir and Naz (2011), Çağman and Enginoğlu (2010), Ali et al. (2014), and Feng et al. (2018). Data indicate that while many soft metric spaces satisfy soft T1T_1T1 and soft T2T_2T2 axioms under defined soft distance functions, soft regularity and Hausdorffness in arbitrary soft topological spaces are inconsistently satisfied. Soft metric spaces (X,d,E)(X, d, E)(X,d,E) are evaluated using soft distance functions d:X×X→[0,∞)Ed: X \times X \to [0, \infty)^Ed:X×X→[0,∞)E, where soft neighborhoods are defined by soft radii across parameters EEE. Quantitative examples show that parameter variations influence satisfaction of soft separation axioms and soft regularity. Soft topological spaces are limited in predictability of axiom satisfaction due to arbitrary soft open and closed set constructions, highlighting variability across parameterized frameworks.

Limitations include reliance on existing secondary data and examples from prior studies, which may not encompass all types of soft topological constructions or parameter sets. Structural behaviors in high-dimensional parameter spaces and non-metric soft topologies remain underexplored, limiting generalizability. The study does not include experimental or empirical generation of new soft spaces; analysis is constrained to documented quantitative examples, soft neighborhood systems, and soft distance functions provided in literature.

DISCUSSION:

Soft set theory provides a parameterized extension of classical set theory, allowing the study of uncertainty through soft topological and soft metric spaces. Data from Shabir and Naz (2011) indicate that soft topological spaces (X,τ,E)(X, \tau, E)(X,τ,E) satisfy soft T0T_0T0 and soft T1T_1T1 axioms in a majority of constructed examples, while soft T2T_2T2 (soft Hausdorff) and soft regularity are satisfied inconsistently depending on the configuration of soft open and closed sets. Quantitative examples show that soft regularity requires, for each soft point xex_exe and soft closed set FFF not containing xex_exe, the existence of disjoint soft open neighborhoods UUU and VVV containing xex_exe and FFF respectively, a condition not universally met in arbitrary soft topologies. Soft metric spaces (X,d,E)(X, d, E)(X,d,E), as analyzed by Çağman and Enginoğlu (2010), generally satisfy soft T1T_1T1 and soft T2T_2T2 axioms under defined soft distance functions d:X×X→[0,∞)Ed: X \times X \to [0, \infty)^Ed:X×X→[0,∞)E. Quantitative data indicate that soft regularity in metric spaces is largely maintained, as soft neighborhoods can be parameterized with soft radii to separate soft points from soft closed sets. Comparative data highlight structural differences: soft metric spaces demonstrate more consistent satisfaction of separation axioms than soft topological spaces due to the inherent structure provided by soft distances. Further literature (Ali et al., 2014; Feng et al., 2018) shows that soft separation axioms and soft regularity interact with other soft topological properties such as soft continuity, soft compactness, and soft connectedness. Data indicate that spaces satisfying soft T2T_2T2 axioms are more likely to support soft continuous mappings and well-defined soft neighborhoods, whereas failure of soft regularity correlates with ambiguous soft closures and non-disjoint neighborhood systems.

RECOMMENDATIONS:

Quantitative data from Shabir and Naz (2011) and Çağman and Enginoğlu (2010) indicate that soft regularity and separation axioms are more consistently satisfied in soft metric spaces than in arbitrary soft topological spaces, suggesting the adoption of soft distance-based structures to enhance compliance with soft T1T_1T1, T2T_2T2, and soft regularity conditions. Parameterized definitions of soft open sets and soft neighborhoods should be standardized across soft topological constructions to improve predictability of axiom satisfaction, as variations in parameter sets EEE have been shown to influence compliance significantly. Further quantitative analysis from Ali et al. (2014) and Feng et al. (2018) indicates that soft neighborhood systems and soft closure operations can be refined to better satisfy soft regularity and soft separation axioms. Structured evaluation of soft points and soft closed sets, combined with disjoint soft open neighborhood definitions, is recommended to ensure soft Hausdorffness and parameterized separation in complex soft topological spaces. Soft metric approaches should be extended to high-dimensional parameter sets, as data demonstrate that increasing EEE dimensions affects neighborhood separation and axiom satisfaction. Comparative data indicate that embedding soft topological spaces within soft metric frameworks improves operational consistency of soft T1T_1T1 and T2T_2T2 axioms, allowing for more reliable applications in soft continuity, soft compactness, and functional soft mappings.

CONCLUSION:

Soft set theory enables parameterized extensions of classical topology and metric spaces, providing a framework to analyze soft regularity and separation axioms under uncertainty. Data from Shabir and Naz (2011) show that soft topological spaces (X,τ,E)(X, \tau, E)(X,τ,E) generally satisfy soft T0T_0T0 and soft T1T_1T1 axioms, while soft T2T_2T2 (soft Hausdorff) and soft regularity are satisfied inconsistently depending on soft open and closed set constructions and parameter variations. Soft metric spaces (X,d,E)(X, d, E)(X,d,E), analyzed by Çağman and Enginoğlu (2010), demonstrate consistent satisfaction of soft T1T_1T1 and soft T2T_2T2 axioms and largely maintain soft regularity under defined soft distance functions d:X×X→[0,∞)Ed: X \times X \to [0, \infty)^Ed:X×X→[0,∞)E. Quantitative analyses from Ali et al. (2014) and Feng et al. (2018) indicate that soft regularity and separation axioms interact with soft continuity, soft compactness, and soft neighborhood systems. Parameter sets EEE and the structure of soft neighborhoods critically influence axiom satisfaction. Comparative data show that embedding soft topological spaces within soft metric frameworks enhances consistency of soft T1T_1T1, T2T_2T2, and regularity properties, facilitating reliable functional mappings and structural analysis. These data collectively indicate that soft metric structures provide more robust compliance with separation axioms and soft regularity than arbitrary soft topological constructions, while parameterized definitions of soft open and closed sets, soft neighborhoods, and soft distances are essential for consistent axiom satisfaction across soft set-based frameworks.

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