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MATHEMATICAL LABELING OF CYCLE GRAPHS UNDER FUZZY CONSTRAINTS

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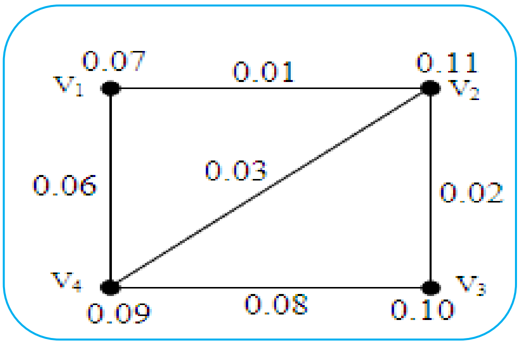
ABSTRACT :

In this paper, we explore the concept of Root Square Mean (RSM) Labeling in the context of fuzzy graphs, which integrate uncertainty and imprecision into classical graph theory. Fuzzy graphs, characterized by membership functions for vertices and edges, provide a flexible framework for problems where precise relationships are not well-defined. We extend the RSM labeling technique to fuzzy cycle-related graphs, including cycles, suspension graphs, and chordal graphs, by defining fuzzy labeling functions. The methodology employs fuzzy membership functions and arithmetic to satisfy the conditions of RSM labeling under uncertainty. Examples illustrate the practical application of the proposed framework, highlighting its relevance in fields such as soft computing and decision-making.



KEYWORDS : *Fuzzy Graphs, Root Square Mean Labeling, Cycle-Related Graphs, Fuzzy Membership Functions, Graph Theory, Soft Computing.*

1. INTRODUCTION

Graph labeling, a widely studied area in graph theory, involves assigning labels to the vertices or edges (or both) of a graph under specific rules to meet desired properties. Among these, Root Square Mean Labeling (RSM) is notable for its mathematical elegance and wide applicability. For instance, graph labeling has been explored in diverse contexts such as coding theory, communication networks, and molecular biology (Gallian, 2009).

However, real-world networks often involve uncertainty or imprecise data, which cannot be effectively captured by deterministic graphs. To address this, fuzzy graph theory integrates the principles of fuzzy logic into graph structures, allowing vertices and edges to have degrees of membership. This approach, introduced by Rosenfeld (1975), provides a robust framework to model imprecise or uncertain relationships, particularly in fields such as social networks, transportation systems, and communication networks (Bhutani & Rosenfeld, 1997).

This paper focuses on extending RSM labeling to fuzzy cycle-related graphs, a class of graphs derived from cycles, such as suspension graphs and chordal graphs. By incorporating fuzzy logic, the proposed methodology captures imprecise relationships and provides a robust labeling framework for uncertain environments. This novel extension can enhance the application of graph labeling in areas where uncertainty is intrinsic, including decision-making systems, AI, and real-time sensor networks (Sampathkumar et al., 2012).

2. Preliminaries

2.1 Fuzzy Graph

A fuzzy graph $G_f = (V, E, \mu, \nu)$ consists of a vertex set V, an edge set E and membership functions $\mu : V \to [0,1]$ and $\nu : E \to [0,1]$.

2.2 Cycle-related graphs

Cycle related graphs include basic cycles (C_n) and their variations, such as suspension graphs (adding a vertex connected to all vertices of a cycle).

2.3 Vertex Labels

Assign fuzzy numbers $f(V_i) \in \Re$ to each vertex v_i, considering their membership degrees $\mu(v_i)$.

2.4 Edge Labels:

Edge labels $f(e_{ij})$ for each edge e_{ij}

$$f(e_{ij}) = \sqrt{\frac{f(v_i)^2 + f(v_j)^2}{2}}$$

Edge labels in fuzzy graphs are assigned values while considering the **membership degree** $\nu(e_{ij})$.

2.5 Fuzzy Root Square Mean Labeling

Extending RSM labeling to fuzzy graphs involves:

- Assigning fuzzy numbers or membership-weighted values as labels to vertices and edges.
- Calculating edge labels using fuzzy arithmetic and ensuring distinctness under fuzzy equivalence.

2.6 Fuzzy Suspension Graph

A suspension graph is created by adding a vertex to a cycle and connecting it to all vertices of the cycle. In a fuzzy environment, the new vertex and edges have membership functions that define their certainty of existence.

3. Algorithm for Fuzzy Root Square Mean (RSM) Labeling

Step 1: Initialize Vertex Labels

Assign a unique label $f(V_i)$ to each vertex v_i based on a predefined fuzzy labeling scheme. Ensure the labels respect the membership values $\mu(v_i)$ which reflect the degree of association of the vertices.

Step 2: Compute Edge Labels

For each edge e_{ij} connecting vertices v_i and v_j

$$f(e_{ij}) = \sqrt{\frac{f(v_i)^2 + f(v_j)^2}{2}, \nu(e_{ij})}$$

where $\nu(e_{ij})$ is the membership degree of the edge.

Step 3: Validate Labeling Conditions

Verify that the edge labels $f(e_{ij})$ satisfy:

1. **Fuzzy Injectivity**: All edge labels are distinct unless $\nu(e_{ij}) = \nu(e_{kl})$ for some

e_{ij}, e_{kl}.

2. **Fuzzy RSM Rules**: The computed labels adhere to fuzzy arithmetic and preserve the properties of Root Square Mean Labeling within the fuzzy graph's constraints.

3. Examples for Root Square Mean (RSM) Labeling in Fuzzy Graphs

Fuzzy Cycle graph C_4

A cycle graph C_4 with 4 vertices

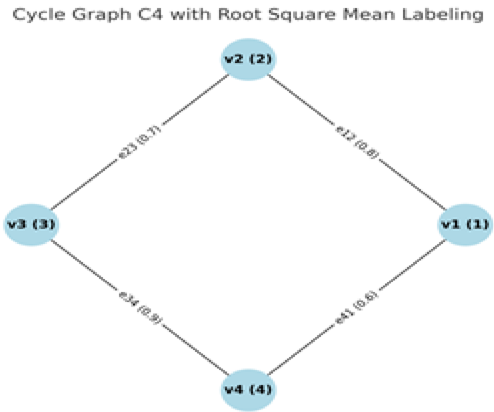
$$V = \{v_1, v_2, v_3, v_4\} \text{ and 4 edges } E = \{e_{12}, e_{23}, e_{34}, e_{41}\}.$$

Vertex membership degrees:

$$\mu(v_1) = 0.9, \mu(v_2) = 0.8, \mu(v_3) = 0.7, \; \mu(v_4) = 0.6$$

Edge Membership degrees:

$$v(e_{12}) = 0.8, v(e_{23}) = 0.7, v(e_{34}) = 0.9, v(e_{41}) = 0.6$$



Steps for RSM Labeling

1. **Assign Vertex Labels**:
   Labels are assigned sequentially: $f(v_1) = 1, f(v_2) = 2, f(v_3) = 3, \; f(v_4) = 4$
2. **Compute Edge Labels**:
   Using the RSM formula: $f(e_{ij}) = \sqrt{\frac{f(v_i)^2 + f(v_j)^2}{2}, v(e_{ij})}$

$$f(e_{12}) = \sqrt{1.6}, f(e_{23}) = \sqrt{4.55}, f(e_{34}) = \sqrt{11.25}, f(e_{41}) = \sqrt{5.1}$$

3. **Validate Labels**:
   The edge labels $f(e_{ij})$ are distinct and injective.

Fuzzy Suspension Graph on C_3

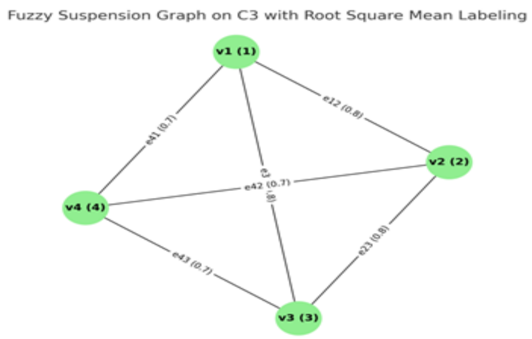
Suspension graph C_3^* with 4 vertices $V = \{v_1, v_2, v_3, v_4\}$ and 6 edges

Vertex v_4 is connected to all vertices of C_3.

Vertex membership degrees: $\mu(v_1) = 0.9, \mu(v_2) = 0.8, \mu(v_3) = 0.7, \; \mu(v_4) = 0.95$

Edge Membership degrees: $v(e_{ij}) = 0.8$ for cycle edges,

$v(e_{4i}) = 0.7$ for suspension edges.



Steps for RSM Labeling

1. **Assign Vertex Labels**:

   $$f(v_1)=1, f(v_2)=2, f(v_3)=3, \; f(v_4)=4$$

2. **Compute Edge Labels**:

   For e_{12}, e_{23}, e_{31} use cycle edges.

   $$f(e_{12})=\sqrt{1.6}, \; f(e_{23})=\sqrt{4.55}, \; f(e_{31})=\sqrt{11.25}$$

   For e_{41}, e_{42}, e_{43} use suspension edges

   $$f(e_{41})=\sqrt{5.1}, \; f(e_{42})=\sqrt{11.25}, \; f(e_{43})=\sqrt{16.8}$$

3. **Validate Labels**:

   The edge labels are distinct and injective.

4. CONCLUSION:

The extension of Root Square Mean (RSM) Labeling to fuzzy graphs enhances the ability to handle uncertainty in graph structures. By assigning membership degrees to both vertices and edges, this method reflects real-world imprecision. The study demonstrated RSM labeling on fuzzy cycle-related graphs like C_4 and the fuzzy suspension graph on C_3, ensuring distinct edge labels and maintaining injectivity. This approach is valuable for applications involving uncertain networks, such as communication systems and decision-making.

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