REVIEW OF RESEARCH

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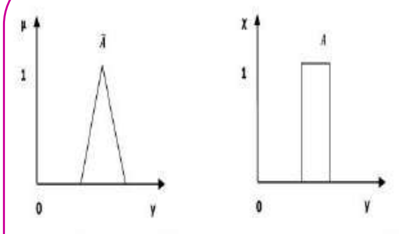
A NEW APPROACH FOR RANKING GENERALIZED FUZZY NUMBER BASED ON WEIGHTED MEAN



Membership Function of Fuzzy set Ã Membership Function of classical set A

Babita Mishra
Assistant Professor ,
Mahatma Gandhi Central University, Motihari, Bihar.
Corresponding Author: Dr. Babita Mishra.

ABSTRACT

Fuzzy set theory deals with uncertain and vague information. In this real world it is very difficult to order fuzzy numbers, So a concept of ranking function begins which can rank or order fuzzy numbers. Ordering of fuzzy numbers plays a key role in various kinds of applied modeling and decision-making process. Several approaches have been made by different authors but some are counterintuitive and not in an incisive manner. Here we propose a new approach for the ranking method which can rank most of the fuzzy numbers and also satisfies our intuition and is discriminating in nature. We easily calculate our proposed method using MATLAB. The method has been validated by some numerical illustrations.

KEY WORDS: *Fuzzy set, Fuzzy subset, Fuzzy Number, Weighted Mean, Ranking.*

1. INTRODUCTION:

Decision-making is most significant in real-life problems such as in medical sciences and engineering. These types of problems incorporate uncertainty and for handling such situations, a fuzzy set plays a vital role. The most crucial aspect of a fuzzy set is that the class of object that belongs to the set has a continuum of grades over [0, 1] [7]. Whereas the crisp sets are those sets whose elements either belong to the set or do not belong to the set.

It is very difficult to order fuzzy numbers because it is defined in form of a set that has various elements with different grades of belongingness. Therefore comparisons and ranking between fuzzy numbers have been the subject of numerous scientific works. However, the majority of the methods have various kinds of drawbacks and restrictions. In particular symmetric fuzzy numbers with the same core and multiple supports, fuzzy numbers with the same support and distinct cores, crisp valued fuzzy numbers with the same support and different heights, and fuzzy numbers with area compensation are not adequately rated in most of the research. To make an approximated conclusion, this necessitates an appropriate ranking procedure to order such ambiguous quantities.

The concept of ordering fuzzy numbers was first proposed by Jain [14,15] with the concept of maximizing set, further Dubois and Prade [6] detailed the same concept. Some techniques have been contrasted and reviewed by Brunelli and Mezei [10], Bortolan and Degani [7], and Wang and Kerre [32,33]. The idea of ranking ambiguous quantities based on the convex combination of the right and left integral value through an index of optimism was first introduced by Kim and Park [9] and has been further generalized by Yu and Dat [24] and Abbasbandy and Asady [17].

Chen[19] has used the concept of maximizing and minimizing set approaches for ranking. By maximizing and minimizing the set to ensure more completeness, Asady [10] and Chou. et.al [12] made an effort to get around the approach constraints. An idea of the centroid indexing method was first proposed by Yager[16], further, Cheng [4,5] has used the distance of centroid points to rank fuzzy numbers. Chu and

Tsao [26] have also worked on the concept of centroid by calculating the area between the centroid point and the origin.

Later on, correction to the centroid formula and a mechanism for ranking generalized fuzzy numbers were given by Wang et.al.[34], this method has been further revised by Chu-Tsao, Wang & Lee [33]. A new approach was proposed by Chen and Chen [19] which was based on different heights and different support of a fuzzy number, further Nasseri et.al.[20] has developed a new approach based on the angle of reference function. There is some drawback to Naseri's and Chen at el.'s approach to ranking. By multiplying two discriminatory parts of the fuzzy number, Nguyen [28] develops a single index and presents comparative reviews.

Based on these ranking techniques, several researchers studied different methodologies and approaches that have been used in various diverse situations such as multi-criteria decision-making, control optimization [4], robot selection [27] supplier selection, logistic center allocation [2,26] facility location determination [21], choosing mining method [1], manufacturing process monitoring [21,2]cutting force prediction, weapon procurement decision [11], Data analysis, etc.

Besides these, there is a wide range of scope for further studies. This study provides a new concept of ranking fuzzy numbers based on a weighted mean value of the left and right fuzziness along with the logarithmic function. Several numerical examples are demonstrated based on primary data and their ranking results are compared with existing methodologies which validate our approach.

2. PRELIMINARIES

This section describes some basic definitions which were introduced by Zimmerman [8] and Lee[33]

2.1 Definition 1 (Fuzzy subset) Let $\mathbb{R}$ be a non-empty set. The fuzzy subset $\tilde{A}$ of $\mathbb{R}$ is defined by a function $\mathcal{M}_{\tilde{A}}: \mathbb{R} \to [0,1]$. $\mathcal{M}_{\tilde{A}}$ is called the membership function.

2.2 Definition 2 ($\alpha - cut\ set$). The α-cut set of $\tilde{A}$, denoted by $\tilde{A}_\alpha$, and is defined by $\tilde{A}_\alpha = \{x \in \mathbb{R} : \mathcal{M}_{\tilde{A}} \geq \alpha\}$ for all α ∈ (0, 1]. The 0-cut set $\tilde{A}_\alpha$ is defined as the closure of a set $\{x \in \mathbb{R} : \mathcal{M}_{\tilde{A}} > 0\}$.

2.3 Definition 3 ($\alpha - level\ set$). The α-level set of $\tilde{A}$, denoted by $\tilde{A}_\alpha$, and is defined by $\tilde{A}_\alpha = \{x \in \mathbb{R} : \mathcal{M}_{\tilde{A}} = \alpha\}$ for all α ∈ [0, 1].

2.4 Generalized Fuzzy Number: A fuzzy number $\tilde{A}$ = (a, b, c, d; ω) is described as a fuzzy subset of the real line $\mathbb{R}$ with a membership function $\mathcal{M}_{\tilde{A}}$ is given as

$$\mathcal{M}_{\tilde{A}}(x) = \begin{cases} \mathcal{M}_{\tilde{A}}^{L}(x), & a \leq x < b \\ \omega, & b \leq x \leq c \\ \mathcal{M}_{\tilde{A}}^{R}(x), & c < x \leq d \\ 0, & otherwise \end{cases} \tag{1}$$

Where $\mathcal{M}_{\tilde{A}}^{L}(x) : [a,b] \to [0,\omega]$ and $\mathcal{M}_{\tilde{A}}^{R}(x) : [c,d] \to [0,\omega]$; $0 \leq \omega \leq 1$ is constant and $\mathcal{M}_{\tilde{A}}^{L}(x), \mathcal{M}_{\tilde{A}}^{R}(x)$ are left and right membership functions which are strictly increasing and strictly decreasing continuous functions on [0, 1].

A number is said to be fuzzy if it satisfies the following properties:

(i) $\tilde{A}$ is normal if there exists an $x \in \mathbb{R}$ such that $\mathcal{M}_{\tilde{A}}$(x) =1; i. e ω=1

(ii) $\mathcal{M}_{\tilde{A}}$(x) is fuzzy convex.

(iii) $\mathcal{M}_{\tilde{A}}$(x) is upper semi-continuous; i. e $\{x \in \mathbb{R} : \mathcal{M}_{\tilde{A}}(x) \geq \alpha\}$ is a closed subset of $\mathbb{R}$ for each α ∈ (0, 1]

(iv) The 0-level set $\tilde{A}_0$ is a closed subset of $\mathbb{R}$"

Remark **. Let $\widetilde{A}$ be a fuzzy number. Then the following statements holds:**
(i) $\tilde{A}_\alpha^L \le \tilde{A}_\alpha^U$ for all α ∈ [0,1]
(ii) $\tilde{A}_\alpha^L$ is increasing for α ∈ [0, 1]
(iii) $\tilde{A}_\alpha^U$ is decreasing for α ∈ [0, 1]

2.5 Image of Fuzzy Number:
The image of a fuzzy number $\tilde{A} = (a, b, c, d: \omega)$ is denoted as $\widetilde{A}' = (-d, -c, -b, -a: \omega)$.

The membership function for the image of a fuzzy number is defined as

$$\mathcal{M}_{\tilde{A}'}(x) = \begin{cases} \mathcal{M}_{\tilde{A}'}^L(x), & -d \le x < -c \\ \omega, & -c \le x \le -b \\ \mathcal{M}_{\tilde{A}'}^R(x), & -b < x \le -a \\ 0, & otherwise \end{cases} \quad (2)$$

Where $\mathcal{M}_{\tilde{A}'}^L(x) : [-d, -c] \to [0, \omega]$ and $\mathcal{M}_{\tilde{A}'}^R(x): [-b, -a] \to [0, \omega]$; $0 \le \omega \le 1$ is constant and $\mathcal{M}_{\tilde{A}'}^L(x), \mathcal{M}_{\tilde{A}'}^R(x)$ are left and right membership functions which are strictly increasing on $[-d, -c]$ and strictly decreasing continuous function on $[-b, -a]$."

2.6 Generalized Trapezoidal Fuzzy Number:
A fuzzy number $\tilde{A}$ is said to be a generalized trapezoidal fuzzy number if it defines its membership as"

$$\mathcal{M}_{\tilde{A}}(x) = \begin{cases} \mathcal{M}_{\tilde{A}}^L(x) = \omega\frac{x-a}{b-a}, & a \le x < b \\ \omega, & b \le x \le c \\ \mathcal{M}_{\tilde{A}}^R(x) = \omega\frac{x-d}{c-d}, & c < x \le d \\ 0, & otherwise \end{cases} \quad (3)$$

2.7 Generalized Triangular Fuzzy Number:
A fuzzy number $\tilde{A}$ is said to be a generalized triangular fuzzy number if it defines its membership as"

$$\mathcal{M}_{\tilde{A}}(x) = \begin{cases} \mathcal{M}_{\tilde{A}}^L(x) = \omega\frac{x-a}{b-a}, & a \le x < b \\ \omega, & x = c \\ \mathcal{M}_{\tilde{A}}^R(x) = \omega\frac{x-d}{c-d}, & c < x \le d \\ 0, & otherwise \end{cases} \quad (4)$$

2.8 Arithmetic operations on fuzzy numbers:
Let the two fuzzy numbers are $X_i = (a_i,\ b_i,\ c_i,\ d_i;\ \omega_i)$ and $X_j = (a_j,\ b_j,\ c_j,\ d_j;\ \omega_j)$, the following are the arithmetic operations defined between them, where $0 \le \omega_i$ and $\omega_j \le 1$

(i) Addition of two fuzzy numbers
$X_i \oplus X_j = (a_i,\ b_i,\ c_i,\ d_i;\ \omega_i) \oplus (a_j,\ b_j,\ c_j,\ d_j;\ \omega_j)$

$$= (a_i + a_j,\ b_i + b_j,\ c_i + c_j,\ d_i + d_j;\ \min\{\omega_i, \omega_j\})$$

(ii) Subtraction of two fuzzy numbers

$X_i \ominus X_j = (a_i,\ b_i,\ c_i,\ d_i;\ \omega_i) \ominus (a_j,\ b_j,\ c_j,\ d_j;\ \omega_j)$

$$= (a_i - a_j,\ b_i - b_j,\ c_i - c_j,\ d_i - d_j;\ \min\{\omega_i, \omega_j\})$$

(iii) Multiplication of fuzzy numbers

$$X_i \otimes X_j = (a_i, b_i, c_i, d_i; \omega_i) \otimes (a_j, b_j, c_j, d_j; \omega_j)$$

$$= (a_i \times a_j, b_i \times b_j, c_i \times c_j, d_i \times d_j; \min\{\omega_i, \omega_j\})$$

(iv) Division of any fuzzy number

$\frac{X_i}{X_j} = (a_i, b_i, c_i, d_i; \omega_i) \oslash (a_j, b_j, c_j, d_j; \omega_j)$

$= (\frac{a_i}{d_j}, \frac{b_i}{c_j}, \frac{c_i}{b_j}, \frac{d_i}{a_j}; \min\{\omega_i, \omega_j\})$

(v) Scalar multiplication of a fuzzy number by 'k'

$$kX_i = \begin{cases} ka_i, kb_i, kc_i, kd_i; & if\ k \geq 0 \\ kd_i, kc_i, kb_i, ka_i; & if\ k < 0 \end{cases}$$

2.9 Fuzziness region's weighted average value :

Definition: Let $\tilde{A}_i = (a_i, b_{i,} c_{i,} d_i : \omega_i)$ be a fuzzy number and the weighted average value of a fuzzy number is denoted by $\chi^L_{\tilde{A}}(x), \chi^R_{\tilde{A}}(x)$ and is defined as

$$\chi^L_{\tilde{A}}(x) = \frac{\int_{a_i}^{b_i} x\mathcal{M}^L_{\tilde{A}_i}(x)}{\int_{a_i}^{b_i} \mathcal{M}^L_{\tilde{A}_i}(x)} dx \quad (5)$$

$$\chi^R_{\tilde{A}}(x) = \frac{\int_{c_i}^{d_i} x\mathcal{M}^R_{\tilde{A}_i}(x)}{\int_{c_i}^{d_i} \mathcal{M}^R_{\tilde{A}_i}(x)} dx \quad (6)$$

Likewise, if $\chi^L_{\tilde{A}'}(x), \chi^R_{\tilde{A}'}(x)$ indicates the weighted mean values of the left and right fuzziness regions of the image A ' = (-d, -c, -b, -a; ω) of the fuzzy number A = (a, b, c, d; ω), then

$$\chi^L_{\tilde{A}'}(x) = \frac{\int_{-d_i}^{-c_i} x\mathcal{M}^L_{\tilde{A}'_i}(x)}{\int_{-d_i}^{-c_i} \mathcal{M}^L_{\tilde{A}'_i}(x)} dx \quad (7)$$

$$\chi^R_{\tilde{A}'}(x) = \frac{\int_{-b_i}^{-a_i} x\mathcal{M}^R_{\tilde{A}'_i}(x)}{\int_{-b_i}^{-a_i} \mathcal{M}^R_{\tilde{A}'_i}(x)} dx \quad (8)$$

3. CALCULATION OF RANKING SCORE:

Let $R_T(\tilde{A})$ be the ranking score of a general fuzzy number A = (a, b, c, d; ω). Then $R_T(\tilde{A})$ is defined as

$$R_T(\tilde{A}) = \frac{1}{2}[\chi^L_A + \chi^R_A]\log(10 + \frac{\omega}{p}).\ ; p \geq 2 \quad (9)$$

Where $p \neq 0$ is a positive integer, As the weighted mean value indicates the position of the fuzzy number on the real axis, the value of p should be greater than 1 showing less importance of height ω to the location of the fuzzy number on the real axis. Here in our proposed method, we take $p \geq 2$ for weighted score computation of numerical example.

Let $\tilde{A}_i$, $\tilde{A}_j$ be two fuzzy numbers. The ranking of fuzzy numbers has the following properties:

(i) If $R_T(\tilde{A}_i) < R_T(\tilde{A}_j)$ then $\tilde{A}_i < \tilde{A}_j$

(ii) If $R_T(\tilde{A}_i) > R_T(\tilde{A}_j)$ then $\tilde{A}_i > \tilde{A}_j$

(iii) If $R_T(\tilde{A}_i) = R_T(\tilde{A}_j)$ then $\tilde{A}_i = \tilde{A}_j$

Numerical examples

Example 1: Let us consider $\tilde{A}_1 = (5, 6, 7; 1)$, $\tilde{A}_2 = (5.9, 6, 7; 1)$ *and* $\tilde{A}_3 = (6, 6, 7; 1)$ are three fuzzy number .

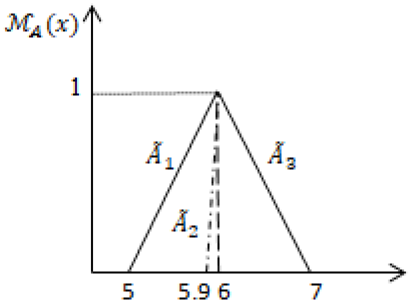


Figure 1: Representation of fuzzy number (Example 1)

Table 1 The ordering outcome obtained for 'example 1' compared by different authors

Authors	Weighted score			Ranking result
	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	
"Abbasbandy&Asady"[17] p=1 p=2	 12 8.375	 12.45 8.8168	 6.5 9.2014	 $\tilde{A}_1 < \tilde{A}_2 < \tilde{A}_3$ $\tilde{A}_1 < \tilde{A}_2 < \tilde{A}_3$
"Asady & Zendehnam "[3]	6	6.225	6.25	$\tilde{A}_1 < \tilde{A}_2 < \tilde{A}_3$
"Abbasbandy& Hajjari"[18]	6	6.075	4.5833	$\tilde{A}_1 < \tilde{A}_2 < \tilde{A}_3$
"Nasseri et.al"[20]	12	12.767	12.854	$\tilde{A}_1 < \tilde{A}_2 < \tilde{A}_3$
"K.Patra" [23]	8	7.5038	7.4571	$\tilde{A}_1 > \tilde{A}_2 > \tilde{A}_3$
Proposed method	6.1271	6.2803	6.4676	$\tilde{A}_1 < \tilde{A}_2 < \tilde{A}_3$

Using our proposed method given example has a high degree of discrimination. From left-to-right spreads of the fuzzy numbers A_1, A_2, and A_3, in set-A, the intuitive and logical order perception will be $\tilde{A}_1 < \tilde{A}_2 < \tilde{A}_3$. The ordering outcomes of the suggested approach are found in Table 1 to be identical to those of rational perception. Abbasbandy & Asady, Asady & Zendehnam, Abbasbandy and Hajjari, Nasseri also deduce the same intuitive ordering result. Only Patra's results do not follow human intuition.

Example 2: Consider a triangular fuzzy number and a trapezoidal fuzzy number given as:

$\tilde{A}_1 = (1,5,5,7;1)$, $\tilde{A}_2 = (1,3,5,9;1)$, the image of these numbers can be obtained as

$\tilde{A}_1' = (-7,-5,-5,-1;1), \tilde{A}_2' = (-9,-5,-3,-1;1)$

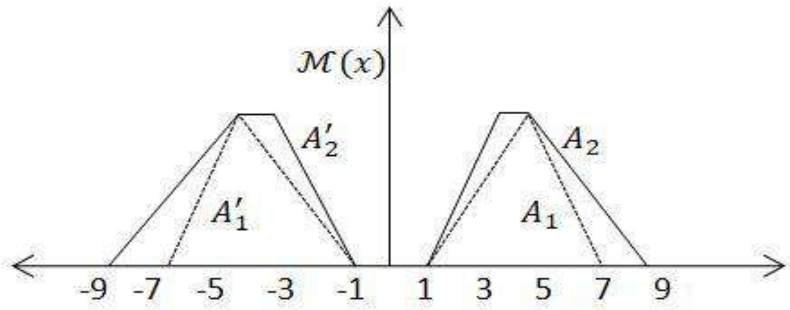


Figure 2: Representation of fuzzy number (Example 3)

Table 2 The ordering outcome obtained for 'example 2' compared by different authors

Author	Weighted score				Ranking result
	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_1'$	$\tilde{A}_2'$	
"Abbasbandy & Asady"[17] P=1 P=2	9.00 6.83	9.00 7.39	-9.00 -6.83	-9.00 -7.39	$\tilde{A}_1' \sim \tilde{A}_2' < A_1 \sim A_2$ $\tilde{A}_2' < \tilde{A}_1' < A_1 < A_2$
"Asady & Zendehnam"[3]	4.50	4.50	-4.50	-4.50	$\tilde{A}_2' \sim \tilde{A}_1' < A_2 \sim A_1$
"Abbasbandy and Hajjari"[18]	4.83	4.17	-4.83	-4.17	$\tilde{A}_1' < \tilde{A}_2' < A_2 < A_1$
"Nasseri et.al"[20], $\lambda = 0.5$	8.62	8.62	-9.38	-9.38	$\tilde{A}_1' \sim \tilde{A}_2' < A_2 \sim A_1$
Chutia and Chutia, $\alpha = 0.5$ [13]	3.58	3.17	-3.58	-3.17	$\tilde{A}_1' < \tilde{A}_2' < A_2 < A_1$
"K. Patra" [23]	2.04	4.50	-2.04	-4.50	$\tilde{A}_2' < \tilde{A}_1' < A_1 < A_2$
Proposed method	6.637	4.850	-4.764	-4.424	$\tilde{A}_1' < \tilde{A}_2' < A_2 < A_1$

Given fuzzy number $\tilde{A}_1$ is a triangular fuzzy number and $\tilde{A}_2$ is a trapezoidal fuzzy number. Similarly $\tilde{A}_1'$ is an image of a triangular fuzzy number and $\tilde{A}_2'$ is an image of a trapezoidal fuzzy number. Although fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ have different cores and right spreads. Due to left or right overlapping of $\tilde{A}_2$ over $\tilde{A}_1$, there is a blurred circumstance for intuition to identify them. Ordering of fuzzy numbers from our proposed method is matched with most of the authors such as Abbasbandy and Hajjari, and Chutia and Chutia for $\alpha = 0.5$. However, some of the authors like Abbasbandy & Asady (p=1), Asady & Zendehnam, and Nasseri fail to order given fuzzy numbers.

Example 3: Consider two fuzzy numbers $\widetilde{A}_1 = (3,5,7;1)$ and $\widetilde{A}_{2=}(3,5,7;0.8)$

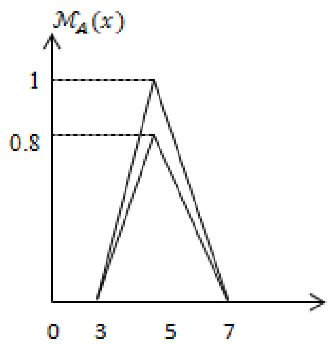


Figure 3: Representation of fuzzy number (Example 3)

The given figure represents the membership function of a fuzzy number. Many authors rank this fuzzy number differently by using their methods. The following results are mentioned below in this table.

Table 3 The ordering outcome obtained for 'example 3' compared by different authors

Author	Weighted score		Ranking Result
	$\tilde{A}_1$	$\tilde{A}_2$	
"Abbasbandy & Asady"[17] P=1 P=2	 10.00 7.257	 10.00 7.257	 $\tilde{A}_1 \sim \tilde{A}_2$ $\tilde{A}_1 \sim \tilde{A}_2$
"Asady & zendehnam"[3]	5.00	5.00	$\tilde{A}_1 \sim \tilde{A}_2$
"Abbasbandy and Hajjari"[18]	5.00	5.00	$\tilde{A}_1 \sim \tilde{A}_2$
"Nasseri et.al"[20]	9.70	9.66	$\tilde{A}_2 < \tilde{A}_1$
" K. Patra" [23]	5.00	4.90	$\tilde{A}_2 < \tilde{A}_1$
"S. Prasad & A. Sinha" [25]	10.53	10.42	$\tilde{A}_2 < \tilde{A}_1$
Proposed method	5.1059	5.0851	$\tilde{A}_2 < \tilde{A}_1$

From the given figure, one can see that both the fuzzy numbers $\tilde{A}_1$and $\tilde{A}_2$ are symmetric at $x = 5$, having the same support but different height. Therefore based on their height human intuition will be $\tilde{A}_2 < \tilde{A}_1$. From the above table, the ordering result of our proposed method is fully matched with our intuition. It is quite clear from the above results that many authors such as Abbasbandy & Asady, Asady & Zenendehnam, Abbasbandy & Hajjari fail to rank given fuzzy numbers. On the other hand, some authors like Patra, Nasseri, S. Prasad & A. Sinha coincide with the intuitive results.

CONCLUSION:

For ranking fuzzy numbers numerous techniques have been implemented by several authors, but no one can rank them accurately. Some are counterintuitive, have less discrimination, and have a lack of inconsistency. To overcome this obstruction, this paper proposed a new ranking technique based on the weighted score method. The proposed method can easily discriminate two fuzzy numbers, support human intuition, and are easy to calculate. The limitation of the other approaches brought on by the compensation of areas can be eliminated by the proposed method. Several numerical examples have been compared using the proposed method. The proposed methodology help to solve the problem of risk analysis, optimization technique, decision-making, and transportation problem.

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