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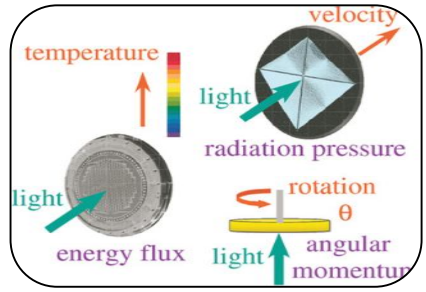
STUDIES ON ASYMPTOTIC FLATNESS AT SPACE LIKE INFINITY AND CONSERVED QUANTITIES



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ABSTRACT

In this paper we define total energy, linear momentum and angular momentum. Then we give an overview followed by a rigorous discussion of how these quantities are conserved.

KEYWORDS: *linear momentum and angular momentum.*

INTRODUCTION :

A 3-manifold H is said to be Euclidean at infinity if there exists a compact set $K \subset H$ such that $H \setminus K$ is diffeomorphic to $R^3 \setminus B$, where B is a ball in R^3. Thus $H \setminus K$ is the domain of a chart.[1-5]

EUCLIDEAN RIEMANNIAN MANIFOLD

According to Euclidean Riemannian manifold (H, g), which is a complete Riemannian manifold which exists a coordinate system (x^1, x^2, x^3) in the complement of K relative to which the metric components $g_{ij} \to \delta_{ij}$ as r .= $\sqrt{\Sigma_{i=1}^{3}(x^i)^2} \to \infty$

PROOF

An asymptotically at initial data set (H, g, k) is an initial data set where (H, g) is an asymptotically Euclidean Riemannian manifold and the components of k approach 0 relative to the coordinate system above as $r \to \infty$.

The fall-off of $g_{ij} - \delta_{ij}$ and k_{ij} with r should be sufficiently rapid for the notions of total energy, linear momentum and angular momentum below to be well defined and finite.

So, the definitions of these notions are the following.

(Arnowitt, Deser, Misner (ADM))

Let $S_r = \{|x| = r\}$ be the coordinate sphere of radius r and dS_j the Euclidean oriented area element of S_r. Then we define

- Total Energy

$$E = \frac{1}{4} \lim_{r\to\infty} \int_{S_r} \sum_{i,j} (\partial_i \bar{g}_{ij} - \partial_j \bar{g}_{ii}) \, dS_j \, , \qquad (1)$$

- Linear Momentum

$$P^i = -\frac{1}{2}\lim_{r\to\infty}\int_{S_r}(k_{ij} - \bar{g}_{ij}\, trk)\, dS_j\ , \qquad (.2)$$

- Angular Momentum

$$J^i = -\frac{1}{2}\lim_{r\to\infty}\int_{S_r}\epsilon_{ijm}x^j(k_{mn} - \bar{g}_{mn}\, trk)\, dS_n\ . \qquad (3)$$

Total energy, linear and angular momentum are conserved quantities at spacelike infinity. We begin the discussion of where these quantities come from. Recall the fundamental theorem of Noether.

Now, in a local coordinate description we can write

L*(x, q, v)

for a Lagrangian. Here x^μ (with μ=1,........., n) are the independent variables, $q^a=u^a(x)$ (with a=1,.........,m) are the dependent variables, whereas $\upsilon^a_\mu = \frac{\partial u^a}{\partial x^\mu}$ (x) denote the first derivatives of the dependent variables. Further, $p^{*\mu}_a = \frac{\partial L^*}{\partial \upsilon^a_\mu}$ is the canonical momentum and

$$T^{*\mu}_\nu = p^{*\mu}_a v^a_\nu - L^*\delta^\mu_\nu \qquad (4)$$

is the canonical stress.

We consider transformations acting only on the domain of the independent variables. Let X^μ be a vectorfield generating a 1-parameter-group of transformations of this domain leaving the Lagrangian form

$$L^*\, d^n x \quad , \quad d^n x = dx^1 \wedge \cdots \wedge dx^n \qquad (5)$$

invariant. Then the Noether current

$$J^{*\mu} = T^{*\mu}_\nu X^\nu \qquad (6)$$

is divergence-free, i.e.

$$\partial_\mu J^{*\mu} = 0\ . \qquad (7)$$

By the divergence theorem we get. If Σ_1 and Σ_2 are homologous hypersurfaces (in particular $\partial\Sigma_1 = \partial\Sigma_2$), then the following conservation law holds.

$$\int_{\Sigma_2} J^{*\mu}\, d\Sigma_\mu = \int_{\Sigma_1} J^{*\mu}\, d\Sigma_\mu\ . \qquad (8)$$

Now we turn to General Relativity.

The Einstein equations are derived from.

Hilbert's Action Principle

$$L = -\frac{1}{4}R \, d\mu_g$$

Thus it cannot be applied directly to Hilbert's principle

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