Review Of Research
Vol. 3 | Issue. 12 | Sept. 2014
Impact Factor : 2.1002 (UIF)
ISSN:-2249-894X
Available online at www.ror.isrj.net

ORIGINAL ARTICLE

FIVE DIMENSIONAL PLANE GRAVETATIONAL WAVES IN BIMETRIC RELATIVITY

L.S. Ladke

Department of Mathematics, Sarvodaya Mahavidyalaya Sindewahi, Distt. Chandrapur. India.

Abstract:

The plane gravitational wave solutions of the field equations $N_i^j = 0$ *in* V_5 *for bimetric relativity are given by* g_{ij} *satisfying* $Q\rho_i^j + P\sigma_i^j = 0$

which further breaks into $\overline{\overline{w}}_1\rho_i^j + \overline{w}_1\sigma_i^j = 0 = \overline{\overline{\phi}}_1\rho_i^j + \overline{\phi}_1\sigma_i^j$,

$$\overline{\overline{w}}_2\rho_i^j + \overline{w}_2\sigma_i^j = 0 = \overline{\overline{\phi}}_2\rho_i^j + \overline{\phi}_2\sigma_i^j$$

KEYWORDS:

Five Dimensional , plane gravitational wave , Bimetric Relativity.

[1] INTRODUCTION

In the paper refer it to [1], for bimetric relativity of Rosen (1973,74), he has obtained the plane wave solutions g_{ij} of the field equations $N_i^j = 0$ in V_5 by reformulating Adhav and Karade's (1994) definition of plane wave in four dimensional space-time V_4 as follows:

DEFINITIONS

A plane gravitational wave g_{ij} is a non-flat solution of the field equations

$$N_i^j = 0 \qquad (i,j = 1,2,3,4,5) \tag{1.1}$$

In an empty region of space-time such that

$$g_{ij} = g_{ij}(Z), \qquad Z = Z(x^i), \quad x^i = u, x, y, z, t \tag{1.2}$$

in some suitable co-ordinate system such that

$$g^{ij}Z,_i Z,_i = 0, \quad Z,_i = \frac{\partial Z}{\partial x^i} \tag{1.3}$$

$$Z = Z(z,t), \quad Z,_4 = 0, \quad Z,_5 = 0 \tag{1.4}$$

where $N_i^j = \frac{1}{2} f^{\alpha\beta}(g^{hj} g_{hi/\alpha})_{/\beta}$

$$N = N_i^j \;, \; K = \sqrt{g/f} \;, \quad g = \det(g_{ij}) \,, \; f = \det(f_{ij})$$

and the bar (/) stands for *f*-covariant differentiation.

In this definition, the signature convention adopted is

$$g_{aa} < 0, \qquad a = 1,2,3,4$$

$$\begin{vmatrix} g_{aa} & g_{ab} \\ g_{ba} & g_{bb} \end{vmatrix} > 0 \,, \quad \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} < 0$$

$$\begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{vmatrix} > 0 \,, \qquad g_{55} > 0, \tag{1.5}$$

(not summed for $a, b = 1,2,3,4$) and accordingly $g = \det(g_{ij}) > 0.$ (1.6)

Though the definition of plane wave used in paper [1] is similar to that of Adhao and Karade (1994), the result (1.6) is contrary to that of Adhao and Karade which is obvious because of the signature difference and odd dimension of the space-time.

The field equations $N_i^j = 0$ then yield

$$N\rho_i^j + M\sigma_i^j = 0 \,,$$

Which is equivalent to

$$\overline{\overline{w}}_1 \rho_i^j + \overline{w}_1 \sigma_i^j = 0 = \overline{\overline{\phi}}_1 \rho_i^j + \overline{\phi}_1 \sigma_i^j$$

Where

$$w = t + z\phi \,, \qquad \phi = \frac{Z,_4}{Z,_5}.$$

$$M = \overline{w} - z\overline{\phi} \,, \quad N = \overline{\overline{w}} - z\overline{\overline{\phi}}$$

$$\rho_i^j = [\phi^2 - 1] g^{hj} \overline{g}_{hi}$$

$$\sigma_i^j = \frac{d}{dZ}[(1-\phi^2) g^{hj} \overline{g}_{hi}]$$

It is observed that the format of mathematical expression derived by Adhao and Karade (1994) in V_4 is retained even in five dimensional space-time V_5 for BR theory of Rosen (1973,74), where at each point of the space-time there are two metrics

$$ds^2 = g_{ij} dx^i dx^i$$

$$and \quad d\sigma^2 = f_{ij} dx^i dx^i$$

In the present paper, same space-time is considered as in the paper [1] but relax the conditions (1.2), (1.3) and (1.5) with assuming

$$Z = Z(y, z, t), \quad Z,_3 \neq 0, \quad Z,_4 \neq 0, \quad Z,_5 \neq 0 \qquad (1.7)$$

we get some interesting result in bimetric theory of relativity.

[2] SOLUTIONS OF FIELD EQUATIONS

From the equations (1.3) and (1.7), we get

$$g^{33}\phi_1^2 + 2g^{34}\phi_1\phi_2 + 2g^{35}\phi_1 + g^{44}\phi_2^2 + g^{45}\phi_2 + g^{55} = 0 \qquad (2.1)$$

Where $\phi_1 = \frac{Z,_3}{Z,_5}, \phi_2 = \frac{Z,_4}{Z,_5}$ (2.2)

which further yield

$$t + \phi_1 y = w_1(Z) \qquad (2.3)$$

$$t + \phi_2 z = w_1(Z) \qquad (2.4)$$

Where w_1 and w_2 are an arbitrary functions of Z.
Differentiating partially (2.3) with respect to y, t and (2.4) with respect to z, t, we obtain

$$Z,_3 = \frac{\phi_1}{M_1} \ , \qquad Z,_5 = \frac{1}{M_1} \tag{2.5}$$

where $M_1 = \bar{w}_1 - \bar{\phi}_1 y$ (2.6)

$$Z,_4 = \frac{\phi_2}{M_2} \ , \quad Z,_5 = \frac{1}{M_2} \tag{2.7}$$

where $M_2 = \bar{w}_2 - \bar{\phi}_2 z$. (2.8)

Differentiating partially (2.6) with respect to y , t and (2.8) with respect to z, t, we obtain

$$M_{1'3} = \frac{N_1}{M_1}\phi_1 - \bar{\phi}_1 \ , \qquad M_{1'5} = \frac{N_1}{M_1} \tag{2.9}$$

Where $N_1 = \bar{\bar{w}}_1 - \bar{\bar{\phi}}_1 y$ (2.10)

$$M_{2'4} = \frac{N_2}{M_2}\phi_2 - \bar{\phi}_2 \ , \quad M_{2'4} = \frac{N_2}{M_2} \tag{2.11}$$

Where $N_2 = \bar{\bar{w}}_2 - \bar{\bar{\phi}}_2 y$ (2.12)

From equations (2.5), (2.7) and (2.9), (2.11) it is observed that

$$M_1 = M_2 = P \ \ (say) \ \ and \ \ N_1 = N_2 = Q \ \ (say) \tag{2.13}$$

Using equations (2.13) the equations (2.5), (2.7) and (2.9), (2.11) can be rewritten as

$$Z,_3 = \frac{\phi_1}{P} \ , \quad Z,_4 = \frac{\phi_2}{P} \ , \quad Z,_5 = \frac{1}{P} \tag{2.14}$$

And $$P_{,3} = \frac{Q}{P}\phi_1 - \bar{\phi}_1 \ , \quad P_{,4} = \frac{Q}{P}\phi_2 - \bar{\phi}_2 \ , \ P_{,5} = \frac{Q}{P} \tag{2.15}$$

Where a bar (-) over a letter means the derivative with respect to Z.

It is to be noted that the expression of various quantities obtained here retain their forms same as in paper [1].

Presuming f_{ij} as Lorentz metric (-1,-1,-1,-1,+1), the f-covariant derivative becomes the ordinary partial derivative and the field equation(1.1) assume the simple form

$$f^{\alpha\beta}(g^{hj}g_{hi,\alpha}),_\beta = 0 \tag{2.16}$$

Which in view of (1.7) becomes

$$f^{33}(g^{hj}g_{hi'3}),_3+f^{44}(g^{hj}g_{hi,4}),_4+f^{55}(g^{hj}g_{hi,5}),_5=0$$

Which further yield

$$Q\{[(\phi_1^2+\phi_2^2)-1]g^{hj}\overline{g}_{hi}\}+P\{[1-(\phi_1^2+\phi_2^2)]g^{hj}\overline{g}_{hi}+[1-(\phi_1^2+\phi_2^2)]g^{hj}\overline{g}_{hi}$$
$$+[1-(\phi_1^2+\phi_2^2)]\overline{g}^{hj}\overline{g}_{hi}-[2(\phi_1\overline{\phi}_1+\phi_2\overline{\phi}_2)g^{hj}\overline{g}_{hi}]\}=0 \qquad (2.17)$$

Equation (2.17) can be put in the form analogous of Adhav and Karade (1994) as

$$Q\rho_i^j+P\sigma_i^j=0 \qquad (2.18)$$

Where

$$\rho_i^j=[(\phi_1^2+\phi_2^2)-1]g^{hj}\overline{g}_{hi}$$

$$\sigma_i^j=\frac{d}{dZ}\{[1-(\phi_1^2+\phi_2^2)]g^{hj}\overline{g}_{hi}\}$$

Substituting the values of P and Q, equation (2.18) reduces to

$$\overline{\overline{w}}_1\rho_i^j+\overline{w_1}\sigma_i^j=0=\overline{\overline{\phi}}_1\rho_i^j+\overline{\phi}_1\sigma_i^j$$

$$\overline{\overline{w}}_2\rho_i^j+\overline{w_2}\sigma_i^j=0=\overline{\overline{\phi}}_2\rho_i^j+\overline{\phi_2}\sigma_i^j \qquad (2.19)$$

Which are again in the corresponding format of Adhav and Karade (1994). However, instead of single equation obtained in the paper [1], we have two equations (2.19) due to postulation of $Z,_3\neq 0,$ $Z,_4\neq 0,\quad Z,_5\neq 0$

CONCLUSION

We conclude that the plane gravitational waves g_{ij} given by (2.18) or (2.19) are the solutions of the field equations in bimetric relativity. If Z is independent of the variable y, then the work regarding plane gravitational waves in five dimensional space – time in paper [1] can be obtained.

REFERENCE

1.Thengane K. D. (2002) Plane wave solutions of the field equations $N_i^j=0$ in five dimensional space – time." The Mathematics Education Vol. XXXVI (2002)

2. Adhav K. S. and Karade T. M. (1994) "On Plane waves in Bimetric relativity" Einstein Foundation International INDIA Vol. 40,1994 .

3.Rosen , N. (1973) “A biometric theory of gravitation”. Gen. Rela. Grav., 4,435.

4. Rosen , N. (1974) “A theory of gravitation” Ann Phys., 84, 455.

L.S. Ladke
Department of Mathematics, Sarvodaya Mahavidyalaya Sindewahi, Distt. Chandrapur. India.