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NEW RESULTS ON GRAPH THEORY AND DETOUR HUMOMETRIC NUMBER

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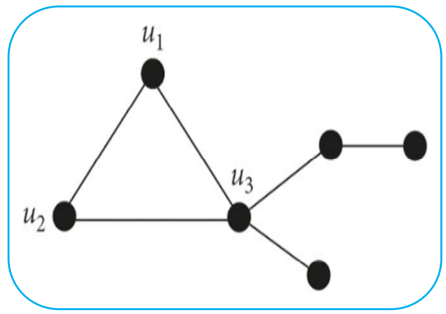
ABSTRACT :

The distance d(x, y) is the length of the shortest way between a couple of vertices x, y ∊ V (G) of an associated chart G = (V, E). Given a subset $S \sqsubseteq V(G)$, the set d(S) is the multiset of pairwise distances between the vertices of S. Additionally, D(S) is the multiset of pairwise detour distances between the vertices of S. Two subsets of the vertex set of a graph G are said to be homometric if their distance multisets are the same. The detour distance D (x, y) is the length of the longest the homometric number of G, denoted by h(G), is the largest integer h for which there are two distinct homometric sets of order h in G.



KEYWORDS : *homometric , X-ray crystallography, hexachordal theorem.*

INTRODUCTION:

Euclidean geometry is where the homeometric set problem in graphs comes from. It was first used in X-ray crystallography and later in DNA restriction site mapping. The fundamental issue under consideration is whether or not a given set of points can be identified from its distances. The issue of deciding homeometric sets likewise shows up in music hypothesis as the hexachordal theorem. The identification of the multiset that corresponds to the multiset of distances realized by a set of points in Euclidean space of a given dimension is just one of many other areas of research in this field.

The idea of homeometric sets in graphs was introduced by Albertson et al. In 2011. In his introductory article, he studied the question, "How large can two disjoint homeometric sets be in a graph?" He defined the homeometric number of a graph *G*; denoted by *h(G)* as the largest integer. h such that two disjoint homeometric sets S_1 are; S_2 in G with $|S_1| = [S_2] = h$.

He proved that every graph on *n* vertices contains homeometric sets of size at least $\frac{c \log n}{\log \log n}$: on the other hand, he only created a class of graphs where the size of homeometric sets cannot exceed $\frac{n}{4}$; In this way, the question of the right-order dimension of the connected ensemble on the maximum size of homeometric sets in graphs is left open. Aksenovich and Ozkahya gave a better lower bound on the size of maximal homeometric sets in trees. They showed that every tree on n vertices contains homeometric sets of size at least $\sqrt[3]{n}$ the bounds later established by Albertson et al. Diameter was modified for two-dimensional graphs and for external planar graphs and trees.

The result presented below establishing the relationship between distance and turn distance for the cycle C_n is well known, but the reference was not found and is therefore proved.

Theorem 1.1: If C_n is the cycle of order n, the *d (x, y) + D (x, y) = n*;

Proof: If C_n is a cycle of order *n*, then it is of size *n*: let x; Let y be any arbitrary pair of vertices in C_n: Since C_n is a cycle, there are two paths P_1 and P_2 of length l_1 and l_2 between x; y: If P_1 is the shortest path, then P_2 is the longest path. And, therefore $l_2 = n - l_1$:

Sets and Graphs of Detour Homometric:

Definition 1.2: Let *G* be a graph and let *S* be a nonempty subset of *V (G)*: the multiset *D(S)* = $\{D(x, y): x; y \in S\}$ element-wise turn distance $x, y \in S \subseteq V(G)$ is called the curve profile of *S*. Two disconnect subsets S_1; $S_2 \subseteq V(G)$ cycles are homeometric if their convolution profiles are equal. Vertigo homeometric number $h_D(G)$; hd is the largest integer; Such that there are two disjoint detour homometric sets S_1, S_2 in a graph *G* with $|S_1| = \lceil S_2 \rceil = h_d$.

By Theorem 1.1, homeometric sets for cycles and trees are also cycle homeometric sets and vice versa. Hence, $h(G) = h_D(G)$ if a graph *G* is a cycle or a tree.

New Result on detour homometric number of a graph:

Proposition 1.2.1: If G is a graph order which is greater than 2 then $1 \le h_D(G) \le \binom{n}{2}$.

Theorem 1.2.2: If *G* is a Hamilton-connected graph of order n; Then $h_D(G) = \left[\frac{n}{2}\right]$.

Proof: Let *G* be a Hamiltonian connected graph with $V(G) = \{v_i : 1 \le i \le n\}$ as its vertex set. G is Hamiltonian connected, $D(v_i; v_j) = n - 1\, for\ all\ v_i, v_j \in V$. Let us consider sets whether the order of G is odd or even $S_1 = \left\{v_1 : 1 \le i \le \left[\frac{n}{2}\right]\right\}$.

Multisets $D(S_1)$ and $D(S_2)$ have the same convolution profile with $\Pi(n-1) = \frac{\binom{n}{2}}{2}$, *$S_1$ and S_2* are cyclic homeometric sets. Further, $S_1 \cap S_2 = \theta\ and\ [V] - |S_1| \lceil S_2 \rceil \le 1$ it follows that $h_D(G) = \frac{n}{2}$.

The preceding theorem identifies an infinite class of nonisomorphic graphs for which $h_D(G) = \left[\frac{n}{2}\right]$. Also, the next theorem has an immediate consequence.

Corollary 1.2.3: Let G be a graph of order n

1. If $G \cong K_n, n \ge 3$; shows the complete graph, then $h_D(K_n) = \left[\frac{n}{2}\right]$.
2. If $G \cong W_n, n \ge 3$, represents the graph of the wheel, then $h_D(W_n) = \left[\frac{n}{2}\right]$.
3. If $G \cong 2$ -leaf connected, $h_D(G) = \left[\frac{n}{2}\right]$. .
4. If G is connected, $h_D(G^3) = \left[\frac{n}{2}\right]$. :
5. If $G \cong 4$-connected planar graph, then $h_D(G) = \left[\frac{n}{2}\right]$:
6. If $G\ is\ 5 - Connected$ *line graph* of degree at least 6, then $h_D(G) = \left[\frac{n}{2}\right]$:

7. If *G is a graph* of order n and size at least $(3^{n-1})h_D(G^3) = \left[\frac{n}{2}\right]$.
8. Let G be a graph of order $n \geq 3$: if $\kappa(G) > \beta_O(G) + 1; Then\ h_D(G) = \left[\frac{n}{2}\right]$.

An *n-barbell* graph is a simple graph obtained by connecting two copies of the complete graph K_n by a bridge. Since K_n is a Hamiltonian connected graph, an n-barbell graph has exactly two Hamiltonian connected graphs, each of order *n* as its induced subgraph. As done in the previous chapter, the idea of an n-barbell graph is generalized by replacing K_n with a Hamiltonian connected graph *n*.

Theorem 1.2.4: If *G* is an n-barbell graph, obtained by connecting two copies of the Hamiltonian connected graph to *n* by bridges, then $h_D(G) = n$.

Proof: If *H* is a connected graph Hamiltonian of order *n* and *G* is a graph obtained by the statement of the theorem, then *G* is of order *2n*: Let H_1 and H_2 be two copies of *H* in *G* such that $V(H_1) = \{u_i : 1 \leq i \leq n\}\ and\ V(H_2) = \{v_i : 1 \leq i \leq n\}$. Without loss of generality, let $u_n v_n \in E(B_n)$ be a bridge in *G*: the set $S_1 = V(H_1)\ and\ S_2 = V(H_2)$ is a turn homeometric are because the multisets $D(S_1)\ and\ D(S_2)$ have the same convolution profile with $\Pi(n-1) = \binom{n}{2}.Since\ S_1 \cap S_2 = \emptyset\ and\ S_1 \cup S_2 = V(G);\ h_D(G) = n.$

A bipartite graph G with partitions *V₁ and V₂* is Hamiltonian lessable, if for any $u \in V_1\ and\ v \in V_2$; is a Hamiltonian path whose terminal vertices are *u* and *v*. The notion of Hamiltonian-lessable graphs is extended as strongly Hamiltonian Iceable graphs.

Theorem 1.2.5: If G is a strongly Hamiltonian Iceable graph of order *n*; Then $h_D(G) = \binom{n}{2}$

Proof: If G is a strongly Hamiltonian laceable graph of order *n*. Then G is a bipartite graph with equal partitions $V_1 = \left\{v_i : 1 \leq i \leq \binom{n}{2}\right\}\ and\ V_2 = \left\{u_i, 1 \leq i \leq \binom{n}{2}\right\}\ and, D(v_i\ v_j) = D(u_i, u_j) = 2n - 2\ and\ D(v_i; u_j) = 2n - 1\ for\ all\ 1 \leq i,j \leq m.$

The sets $S_1 = V_1\ and\ S_2 = V_2\ are\ maximum\ detour\ homometric\ since\ S_1 \cap S_2 = \emptyset, S_1 \cup S_2 = V(G)\ and\ D(V_1)\ and\ D(V_2)$ are multisets with the same convolution profile with $\Pi(2n-2) = n2$: $hence\ h_D(G) = \binom{n}{2}$.

Although the turn homeometric number of a complete bipartite graph has been determined, the determination of the turn homeometric number of a complete tripartite graph is an open problem.

CONCLUSION:

A newly introduced graph parameter cycle homeometric number $h_D(G)$ is tested for the class of graphs with at least one cycle. It is open to investigate the homeometric number of turns of many other graph classes. Determining the homometric number of turns of derivative graphs like complements, line graphs, aggregate graphs etc. also seem promising for future investigation.

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